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BINARY IMAGE DATA COMPRESSION AND ENCRYPTION APPROACH USING ARITHMETIC CODING AND TRAPEZOID METHOD FOR CONTOUR APPROXIMATION

[1] ALI ABDRHMAN UKASHA, [2] MOHAMMED ABDULLAH S. HASSAN, [3]MOHAMED NURI HUSSIN

Electrical and Electronics Engineering Department, Sebha University - Sebha, Libya
Email: [1]ali.ukasha@sebhau.edu.ly, [2]mohamedalfakhri@gmail.com, [3]mohamed.hussin@sebhau.edu.ly

ABASTRACT

This paper presents a new algorithm of image compression that reduces number of bytes required to represent images which are useful in those applications where original image can be retrieved without distortions. Arnold transform is applied for scrambling data to prevent eavesdropping and jamming, reduce interference; increase immunity from various noise and multipath distortions. This work deals with data storage reduction and high-speed transmission in digital communication systems. In the proposed scheme, contour data is compressed with both lossless and lossy techniques using Arithmetic coding (source coding) and Trapezoid contour approximation method respectively. In channel coding we choose the Trellis diagram as convolutional coding. We are going to choose a method depending on one of the spread spectrum technique types which is direct sequence spread spectrum (DSSS). The experimental results are conducted and compared against M-QAM to find the better approach under various performance measures such as Mean Square Error (MSE), Compression Ratio (CR), Normalized Correlation coefficient (NCC), and Signal to Noise Ratio (SNR). Experimental results show that the contours of the original binary image can be compressed by using arithmetic coding and Trapezoid method exceeded to 85% and 17% respectively. The NCC values for the reconstructed binary image exceed to 0.97 for some images. Options for double compression mode, make the proposed scheme attractive for wireless communications systems. The experiments show that the analyzed algorithm reconstructs the image contours exactly with NCC equals to 1. The main advantage of the analyzed digital communication system algorithm is that it uses only few contour points (compressed) instead of huge number of binary images which can be reconstructed perfectly at the receiver with high quality where NCC values approach to 1 in some binary images. This analyzed method is very useful for real time application.

Keywords: trapezoid method; arithmetic coding; arnold transform map; contour extraction; channel coding, QAM modulation; spread spectrum

1. INTRODUCTION

This paper deals with transmit contours points rather than binary images. Contour representation and compression are required and very useful in many applications such as medical images, topographic, and computer vision. The contours of binary image is extracted using method known as single step parallel contour extraction method “SSPCE” [1]. A lot of techniques for polygonal approximation have been proposed in the literature and it can be classified either as sequential method such as in [2], or as split-and-merge approaches such as in [3-5]. The well-known contour description approaches are Cartesian, Polar representations, and Freeman’s (also generalized) chain coding which are useful in some applications [6, 7]. In this paper a fast algorithm for contour compression of trapezoid method is used for contour compression [8].

The aim of this paper is to send encrypted compressed contours data for the binary image with a complete confidentiality using integer Arithmetic coding, Trapezoid method for contour approximation, Spreading spectrum, Arnold transform, and M-Quadrature Amplitude Modulation. The main goal at the receiver is to reconstruct the original binary image completely without any distortion.

2. SINGLE STEP PARALLEL CONTOUR EXTRACTION (SSPCE) ALGORITHM

The object contours are extracted using different window sizes such as a 3x3 pixels which has been used in this work to find the all possible edge directions which connects the central pixel with one of the other remaining pixels surrounding it. To distinguish all four or eight possible line segments connecting nearest neighbors, the directional Freeman chain coding scheme are performed [7, 9, 10]. This approach is faster than that by using of OCE algorithm for the same extracted contour points based on 4/8-directional Freeman chain coding scheme [7].

The object contour edge is a straight line connecting two neighboring pixels which have both a common neighboring object and underground pixels [11]. The edges cannot be extracted according to the following three cases: First if all nine pixels are object pixels (the window is inside an object region). Second, if all nine pixels are background pixels (the window is inside a background region). Third, if the center pixel is an object pixel surrounded by background pixels (it is most probable that the center pixel in this case is a point noise caused by image digitalization).

3. TRAPEZOID METHOD

The idea behind is based on that the contour is represented as a polygon shape [12-14]. The edges points are connected before coded the contour points using Freeman chain coding by a tracing operation. This paper uses method explained in the previous section which known as (SSPCE) for closed contour extraction from binary image. The contour point's segmentation is the main idea to determine four points of the trapezoid shapes. Starting point (SP) and ending point (EP) is the first and last points of the contour segment respectively. Figure 1 illustrate the block diagram of the main idea by determining the ratio between the perpendicular distance (*dB*) of the second point from the diagonal line (line between the stating and *C* points) and the perpendicular distance (*dC*) of the third point from the diagonal line (line between the *B* and ending points) for each segment using the following equation:

$$(dB / dC) < th \qquad \text{Eq. (1)}$$

where *th* is given threshold value.

These values are calculated using simple trigonometric formula. If the equation (1) is not valid the second, third and ending points are stored and the *SP* is shifted to the *EP* of the trapezoid, then a new segment is drawn. Otherwise the third and ending points of the trapezoid are stored and the *SP* is shifted to the *EP*, then a new segment is drawn. These stored points known as the vertices of an edge of the contour approximating polygon. The idea of the analyzed algorithm is illustrated in the block diagram as shown in Figure. 1. Only the co-ordinates of the starting point of the contour segment, and the last processed point are stored. The contours can be described in Cartesian representation, the x and y co-ordinates, or polar representation, using a length of the line *l* from one point to the next in sequence and the angle α between every two lines [12, 14].

4. ARIITHMETIC CODING

The aim of source coding is to take the source data and make it smaller in size. Entropy encryption is one of the most common methods of lossless compression of data, which represents source codes in the least compressed form by a number of variable-length bits. One of the most well-known methods of entropy encryption is the Huffman method and arithmetic coding [15].

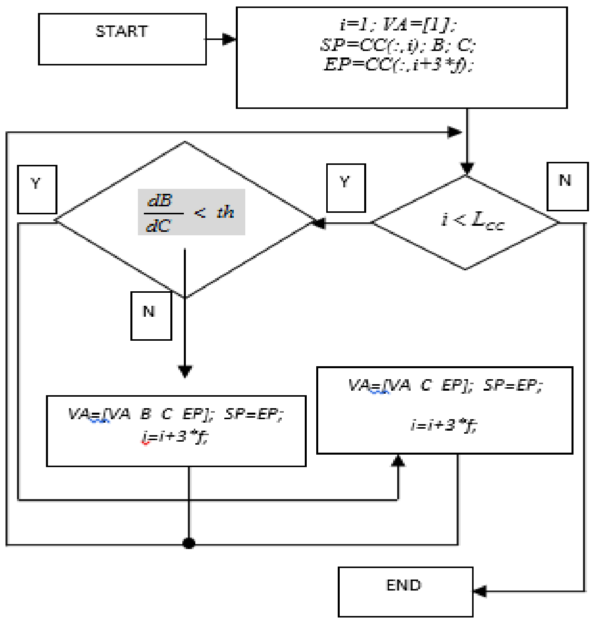


Figure.1 Flowchart for Trapezoid method

Where *VA* is sequence of indices of the final vertices, *CC* is sequence of the input for the contour, *SP* is starting point, *EP* is ending point, *dB*, *dC* and th are as mentioned before (see Figure.1 and equation (1)), L_{CC} is the input contour length, and *f* is the length between each two points of the trapezoid.

The representation of each code by the probability that it will appear in periods of real numbers between 0 and 1 is the main idea of arithmetic coding which are used in this work. Behined that the more probability symbols represent fewer bits and the less likely symbols represent more bits. At the beginning and before doing any coding the message period should be in [0, 1] interval. Proportional to the probability of the corresponding symbol in the message, the range for any period is divided into many sub-periods in a size. The duration of the next symbol is then can derived from the sub-period of the code of the previous symbol code and the same thing is done for the remain codes to the end [16].

5. CHANNEL CODING

In channel coding, the purpose is to find codes which transmit quickly, contain many valid code words and can correct or at least detect many errors such as Reed-Solomon code, Turbo code, Cyclic code, and Convolution code which are using in this research. Convolution coding make every codeword symbol be the weighted sum of the various input message symbols. Maximum-likelihood soft-decision decoded with reasonable complexity using time-invariant trellis, whereas, classic block codes are generally represented by a timevariant trellis and therefore are typically hard-decision decoded (used in this work) [17-19]. A coding scheme for the AWGN channel is based on two parameters. Signal-to-noise ratio (SNR) in decibels (dB) and Spectral efficiency in bits per second per Hertz (b/s/Hz). The Viterbi algorithm is used in many applications and it's idea based on finding the most likely sequence of hidden states and the generated sequence of states is called the Viterbi path (cf. Ref. [20, 21]). Convolutional codes have memory that uses previous bits to encode or decode following bits.

The trellis provides a good framework for understanding decoding. If for instance we have the entire trellis in front of us for a code, and now receive a sequence of digitized bits (or voltage samples). If there are no errors (i.e., the noise level is low), then there will be some path through the states of the trellis that would exactly match up with the received sequence. That path (specifically, the concatenation of the encoding of each state along the path) corresponds to the transmitted parity bits. From there, getting to the original message is easy because the top arc emanating from each node in the trellis corresponds to a "0" bit and the bottom arrow corresponds to a "1" bit. When there are errors, we finding the most likely transmitted message sequence is appealing because it minimizes the bit error rate (BER). At the receiver, we have a sequence of voltage samples corresponding to the parity bits that the transmitter has sent. For simplicity, and without loss of generality, we will assume one sample per bit. If we decode the received bit sequence, the decoding process is termed hard decision decoding ("hard decoding"). If we decode the voltage samples directly before digitizing them, we term the process soft decision decoding ("soft decoding").Applications of the Viterbi algorithm include decoding convolutional codes in Telecommunication (specifically in CDMA, GSM, satellite and other technologies that use digital coding/decoding of signals) as well as applications in speech recognition, speech synthesis, speech enhancement and other technologies. A simple diagram of rate code (*r=k/n*) is shown in Figure 2. Where k is the number of message sympols, and n is the number ofcodeword sympols.

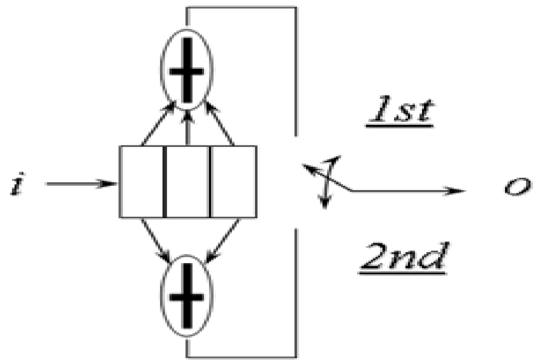


Figure. 2 A simple rate ½, v = 3 convolutional encoder

6. ARNOLD TRANSFORM

'Vladimir I' introduced arnold transform. For an N×N image, two-dimensional Arnold transform is defined as in equation (2). The classical Arnold transformation converts any linear second order ordinary differential equation (LSODE) into the free Galilean particle equation [22]. Therefore, the security of image is strengthened.

Many different approacches like Fass Curve, Gray Code, Arnold transform can be used for image scrambling [23]. The image is returned back once more to the original case after determine number of iterations (m) called 'Arnold Period' using the equation (2); and that is gives the Arnold transform a special property.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \left[\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} \right] \mathrm{mod}(N) \qquad \text{Eq. (2)}$$

where (x, y) and (x', y') are the pixel coordinates of the original image and the encrypted image, respectively.

Arnold transform has a property that the original image will appear when the equation (2) is iteratively calculated m times. The periodicity of Arnold transform (m), is dependent on size of given image. The periodicity value $m \leq N^2/2$ and some specific values under different image sizes N are listed in Table I [24].

Table. 1 The periodicity values m under different image sizes N

N	60	100	120	128	256	480	512
m	60	150	60	96	192	240	384

7. SPREAD SPECTRUM

In spread spectrum, we combine signals from different sources to fit into a larger bandwidth, but our goals are to prevent eavesdropping and jamming. To achieve these goals, spread spectrum techniques add redundancy.

After the signal is created by the source, the spreading process uses a spreading code and spreads the bandwidth. The spreading code is a series of numbers that look random, but are actually a pattern.

There are two common techniques to spread the bandwidth refereed to Frequency hopping spread spectrum (FHSS) and Direct sequence spread spectrum (DSSS) which is used in this work.

The direct sequence spread spectrum (DSSS) technique also expands the bandwidth of the original signal, but the process is different. In DSSS, we replace each data bit with n bits using a spreading code. In other words, each bit is assigned a code of n bits, called chips, where the chip rate is n times that of the data bit.

The characteristic of this technology can be determined as follows: Several users can share same higher bandwidth with little interference; Immunity from various noise and multipath distortion; and can hide/encrypt signals [25]. Figure 3 illustrate the general block diagram for spread spectrum system.

In direct sequence spread spectrum (DSSS) each bit represented by multiple bits using spreading code. Spreading code spreads signal across wider frequency band. One method can combine input with spreading code using XOR where input bit 1 inverts spreading code bit and input zero bit doesn't alter spreading code bit. Data rate equal to original spreading code. Performance similar to FHSS [26]. DSSS is used in this paper and the block diagram for its transmitter and receiver is shown in Figure 4 and Figure.5 respectively.

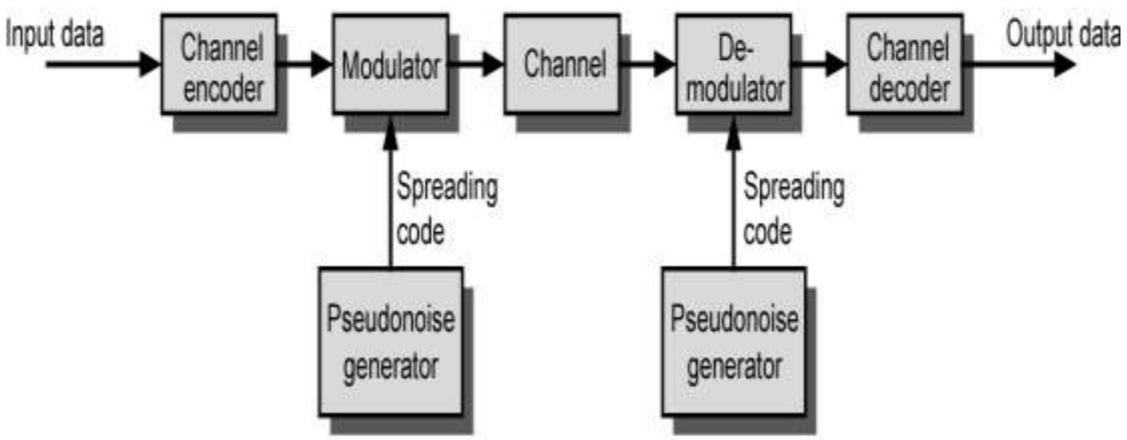


Figure. 3 General model of spread spectrum system

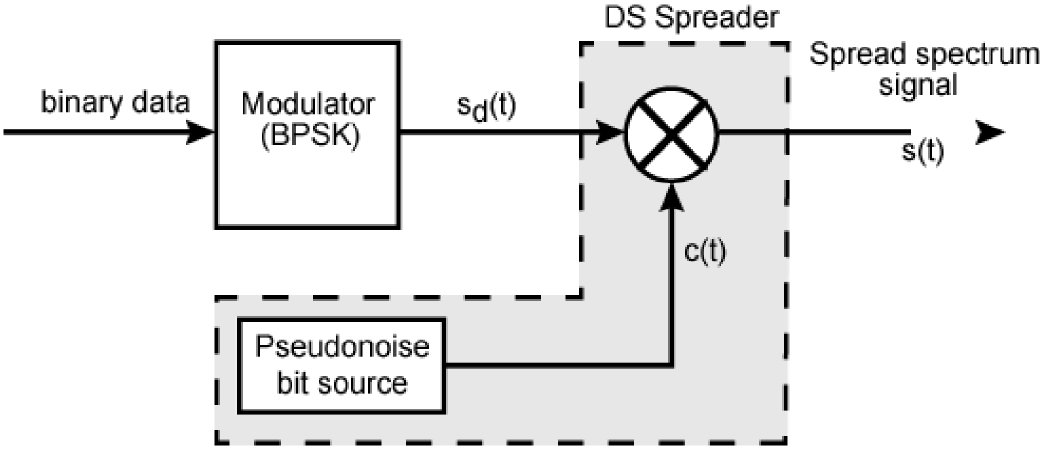


Figure. 4 DSSS Transmitter

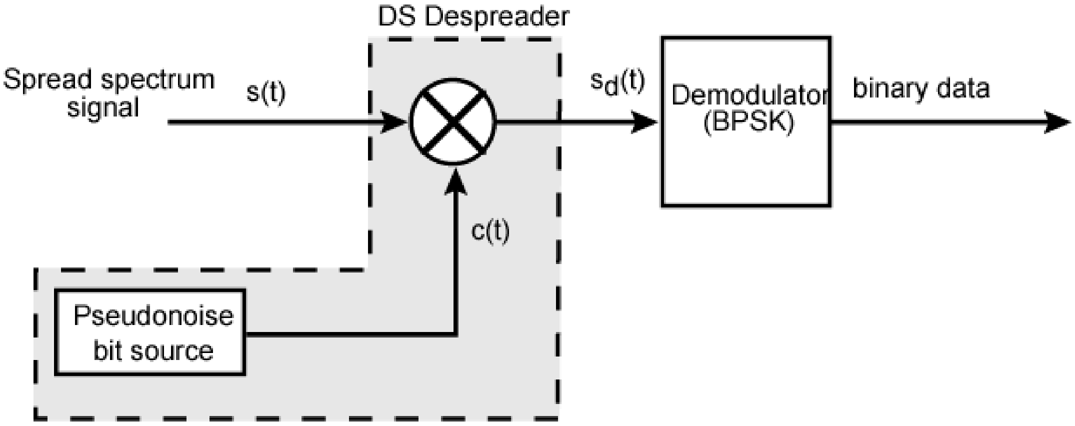


Figure. 5 DSSS receiver

8. THE PROPOSED ALGORITHM

This section illustrates the main stages of the analyzed algorithm. The method of security improvement based on Arnold transform to ensure secure transmission of data. Procedures of encoding and decoding are also discussed. Figure 6 shows the block diagram of the embedding procedure.

The encoding process can be described by the following steps:

1. Read binary image.
2. Contour extraction using SSPCE method.
3. Contour compression in spatial domain using Trapezoid method for contour approximation.
4. Applying Arnold transform for security encoding.
5. Source coding using Arithmetic lossless coding.
6. Channel convolutional coding using Trellis diagram.
7. Applying different M-QAM modulation.
8. Direct spread spectrum (DSS).
9. Additive white Gaussian noise (AWGN).

The decoding process is an invertible of the transmission process except that the Viterbi algorithm are used for detection in channel decoding stage.

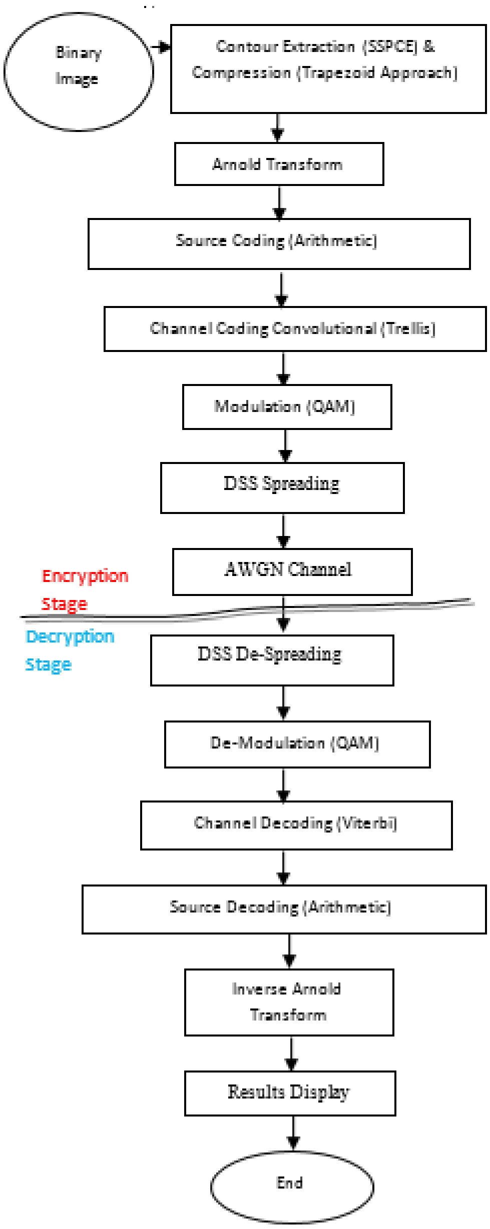


Figure. 6 Flowchart of the proposed algorithm

9. EXPERIMENTAL RESULTS AND DISCUSSIONS

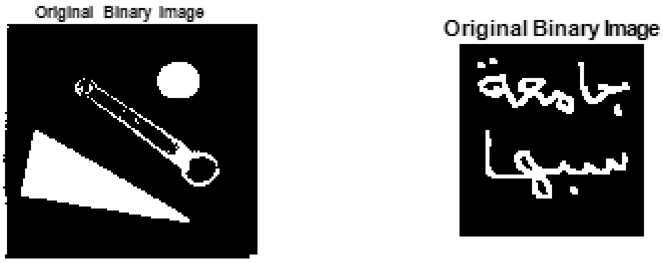
The performance of data hiding scheme was tested on several images commonly used for this purpose. MATLAB (2016) programming has been used to obtain all results. The specifications of the computer which are used to obtain the results are: Intel (R) Core (TM) i7 – 3537U CPU @ 2.00GHz 2.50 GHz. The images used in this paper are binary images. Tested image are 256x256 ('Tools') and 160x160 ('Arabic Text') sizes and are shown in Figure 7.

There are various ways to evaluate how much correlation between the received extracted contours and the original contours of the binary image. The normalized cross correlation (NCC) is one of the well-known measurements that used for this purpose. The normalized cross correlation defines as in equation (3) [26].

$$NCC(A,B) = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} A(i,j)B(i,j)}{\sum_{i=1}^{m}\sum_{j=1}^{n} A(i,j)^2} \qquad \text{Eq. (3)}$$

where *A(i, j)* and *B(i, j)* are the original and reconstructed images respectively.

The value of NCC is between, $0 \leq NCC \leq 1$. In ideal cases this value should be equal to 1 [26]. The closest values to 1 are desirable where those images approach to each other.



(a) (b)

Figure. 7 Binary image: (a) tools, and (b) Arabic text

- The normalized correlation coefficient (*NCC*) was adopted to evaluate the perceptual distortion of the proposed scheme. The visual quality measures of the reconstructed binary image are listed in Table II and Table III for tested images; and it can be notice that the *NCC* value effected with image texture, in case of images with low texture for instance 'Tools' the *NCC* values will be larger than that for images with high texture contents such as 'Arabic Text'. The perceptual requirements are satisfied in the tested images with *NCC* greater than 0.8 without significant distortions in the reconstructed image. The original binary images can be precisely reconstructed at the receiver with *NCC* for 'Tools', and 'Arabic Text' are 0.9753, and 0.9498 respectively.
- The proposed algorithm can reconstruct exactly the contours images with *NCC* equals to one for both test images.
- From Tables II and III, it is clear that the contours of the images are compressed using arithmetic coding/Trapezoid approximation exceeds to 85% and 17% for ‘Tools’ image and 87% and 14% for ‘Arabic Text’ image respectively. The additive white Gaussian noise carried by 16-QAM is much than 32 and 64-QAM.
- Using Arnold transform for scrambling image can improve the security data system since using private key to transmitted data.

To receive the data with accepted level of distortion a lot of experiments is done for different values of additive white Gaussian noise, otherwise the received image is not accepted with low value of NCC as shown in Figure 8.

Table. 2 Tools image results

QAM Modulation	AWGN (dB)	NCC Contours	NCC Binary	Consumed Time (s)	Arithmetic/ Trapezoid Compression
16	16.40	1.0000	0.9753	20.9063	85.1227%
32	20.50			25.6563	17.4067%
64	25.00			23.1875	

Table. 3 Arabic text image results

QAM Modulation	AWGN (dB)	NCC Contours	NCC Binary	Consumed Time (s)	Arithmetic/ Trapezoid Compression
16	16.50	1.0000	0.9498	6.5781	78.0938%
32	19.40			7.7188	
64	24.00			6.6406	14.7760%

where *NCC* Contours is the reconstructed image contours; and NCC Binary is the reconstructed binary image.

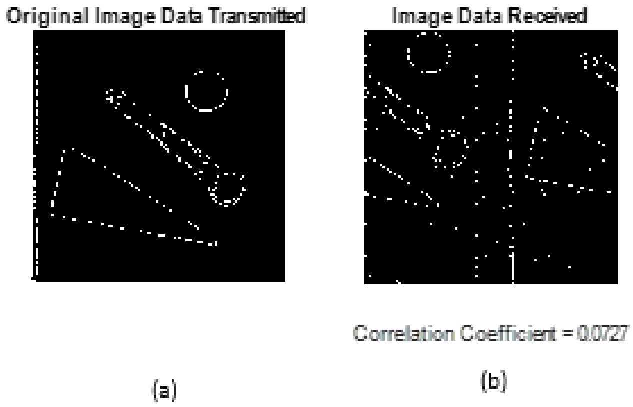
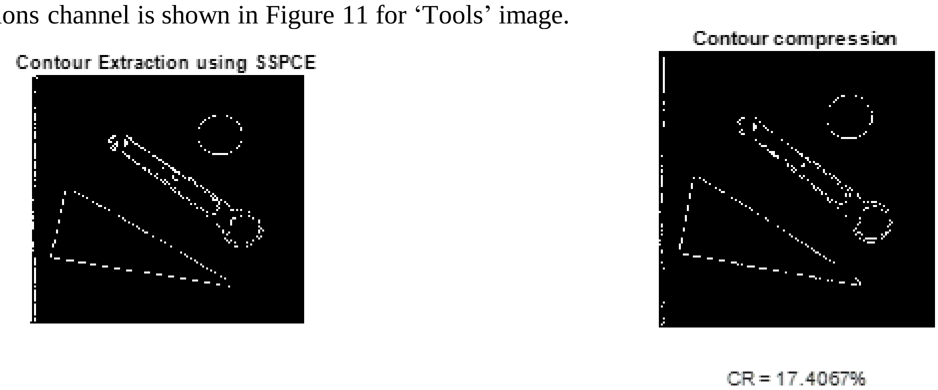


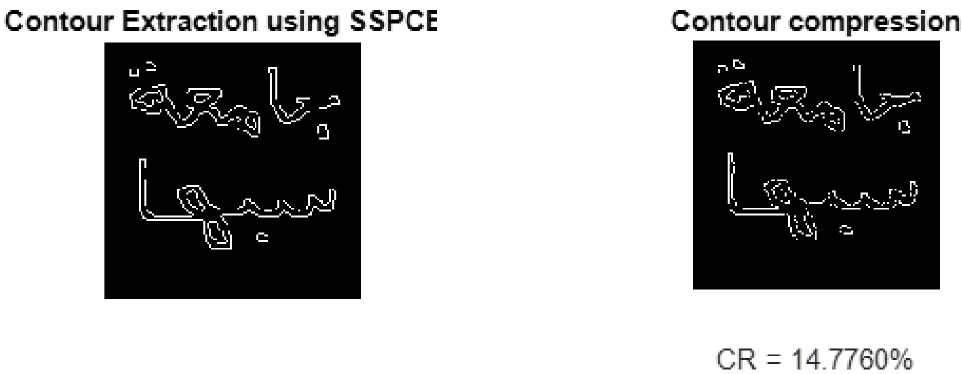
Figure. 8 Tools image: (a) Original data contours compression (1395 contour points), and (b) Data contours compression received (1395 contour points)

Figure 9 and Figure 10 shows the results of applying contours extraction and contour compression using SSPCE method and Trapezoid algorithm for 'Tools' and 'Arabic Text' images respectively. The contours compression images are scrambling using Arnold transform using private key before data transmission via communications channel is shown in Figure 11 for 'Tools' image.



(a) (b)

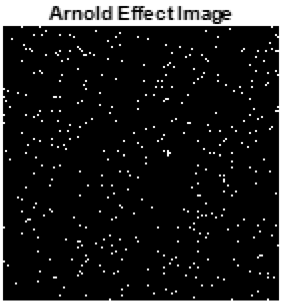
Figure. 9 Tools image: (a) Contours extraction using SSPCE method (1689 contour points), and (b) Contour compression using Trapezoid method (1395 contour points)



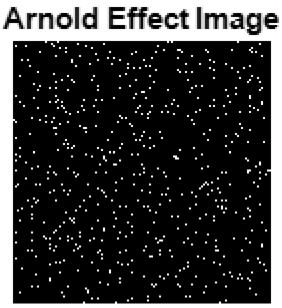
(a) (b)

Figure. 10 Arabic Text image: (a) Contours extraction using SSPCE method (1049 contour points), and (b) Contour compression using Trapezoid method (894 contour points)

Figure 12 and Figure 13 shows the results for each stage of the analyzed algorithm by using of 16-QAM modulation for test images. The contours of the images can be reconstructed at the receiver exactly by NCC=1 as shown in Figure 14.

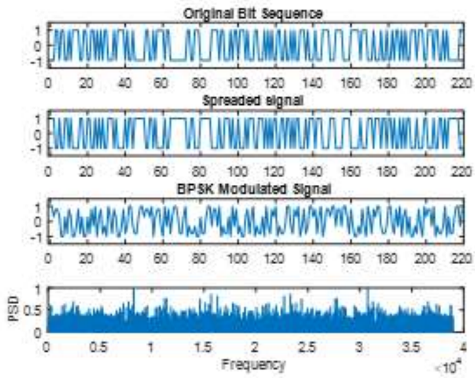


(a)

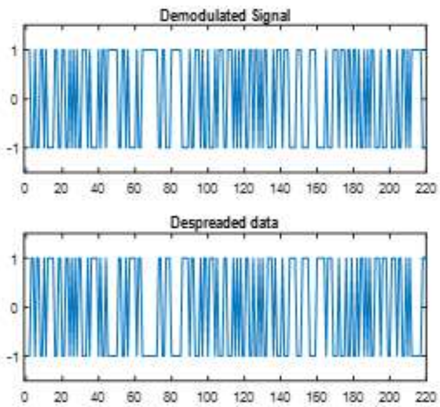


(b)

Figure. 11 Effect of Arnold transform for (a) Tools contours image (1395 contour points), and (b) Arabic Text image (894 contour points)



(a)



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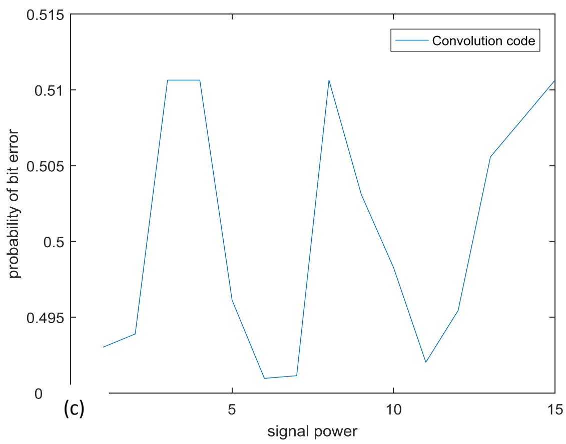

(c)

Figure. 12 Tools image using 16-QAM: (a) Spreading spectrum and modulating processes, (b) Demodulaing and despreading spectrum processes, and (c) Probapility bit error versus signal power using Viterbi decoding detector

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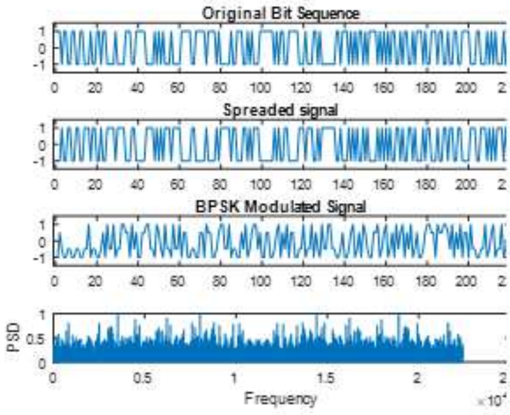
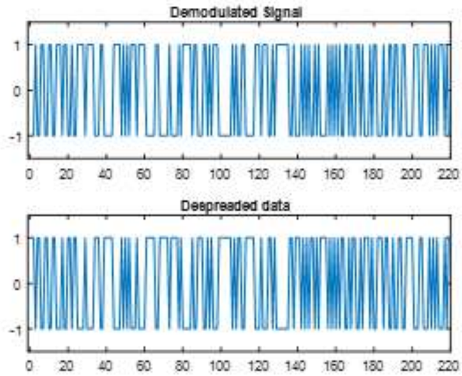


(a)



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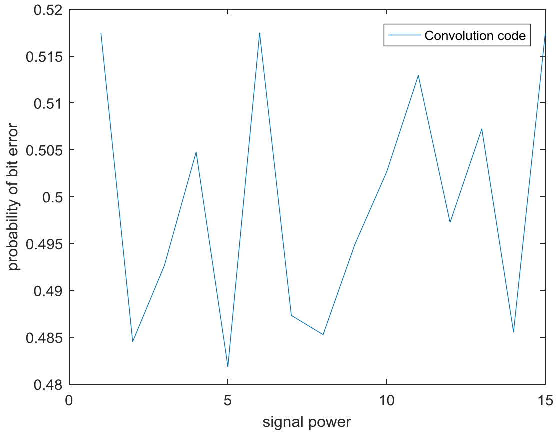
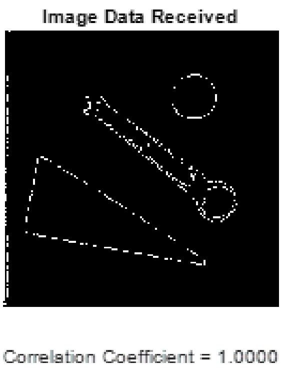
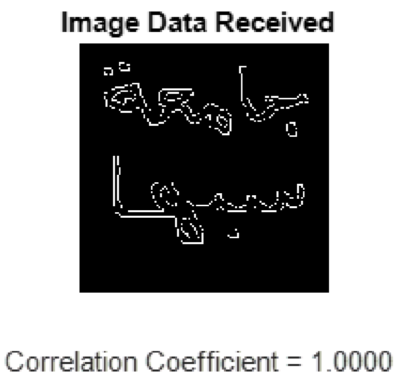


(c)

Figure. 13 Arabic Text image using 16-QAM: (a) Spreading spectrum and modulating processes, (b) Decmodulaing and despreading spectrum processes, and (c) Probapility bit error versus signal power using Viterbi decoding detector



(a)



(b)

Figure. 14 Image contours reconstruction for (a) Tools image, and (b) Arabic teaxt image

The original binary images can be reconstructed at the receiver with high quality as shown in Figure 15. To detect the signal data the Viterbi decoding algorithm with hard decision is used at the receiver. Figure 16 and Figure 17 shows the probability of bit error versus signal power using 16, 32, and 64-QAM for 'Tools' and 'Arabic Text' images respectively.

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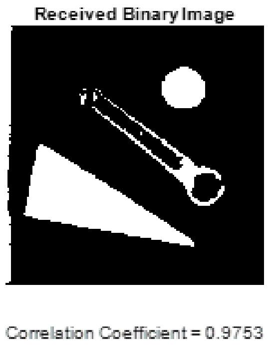
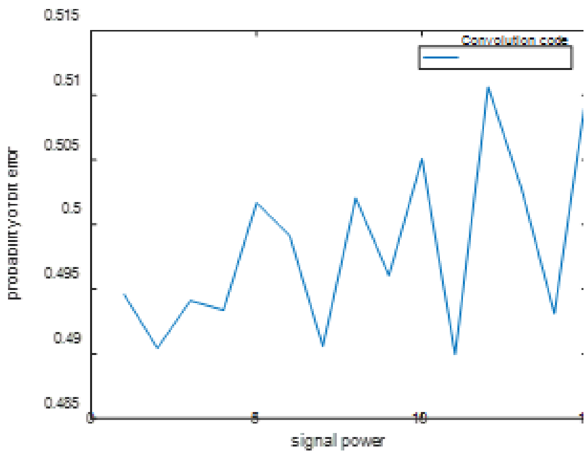


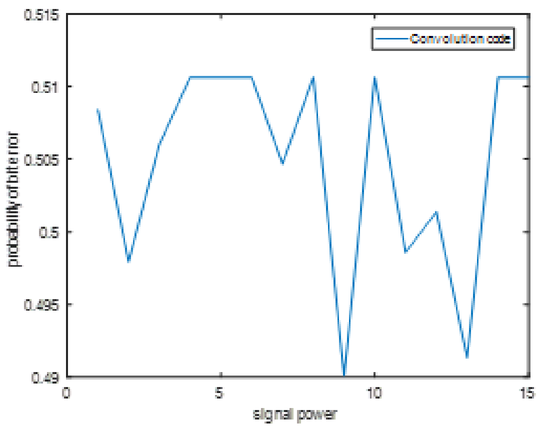
(a)

(b)

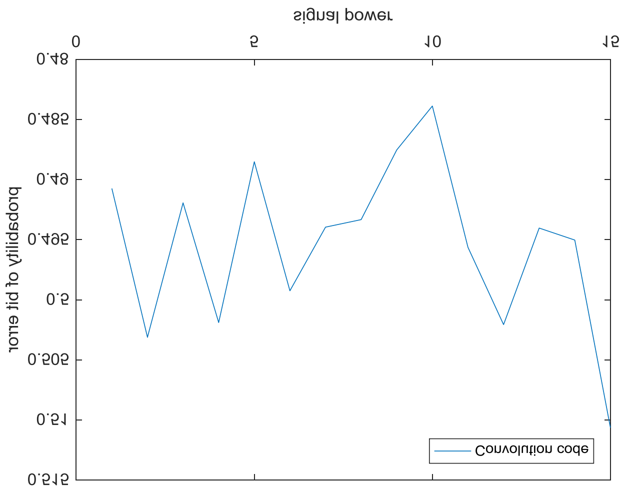
Figure. 15 Original binary images reconstruction for (a) Tools image, and (b) Arabic Text image



(a)



(b)



(c)

Figure. 16 'Tools' contour data detection using Viterbi decoding algorithm for (a) 16-QAM, (b) 32-QAM, and (c) 64-QAM

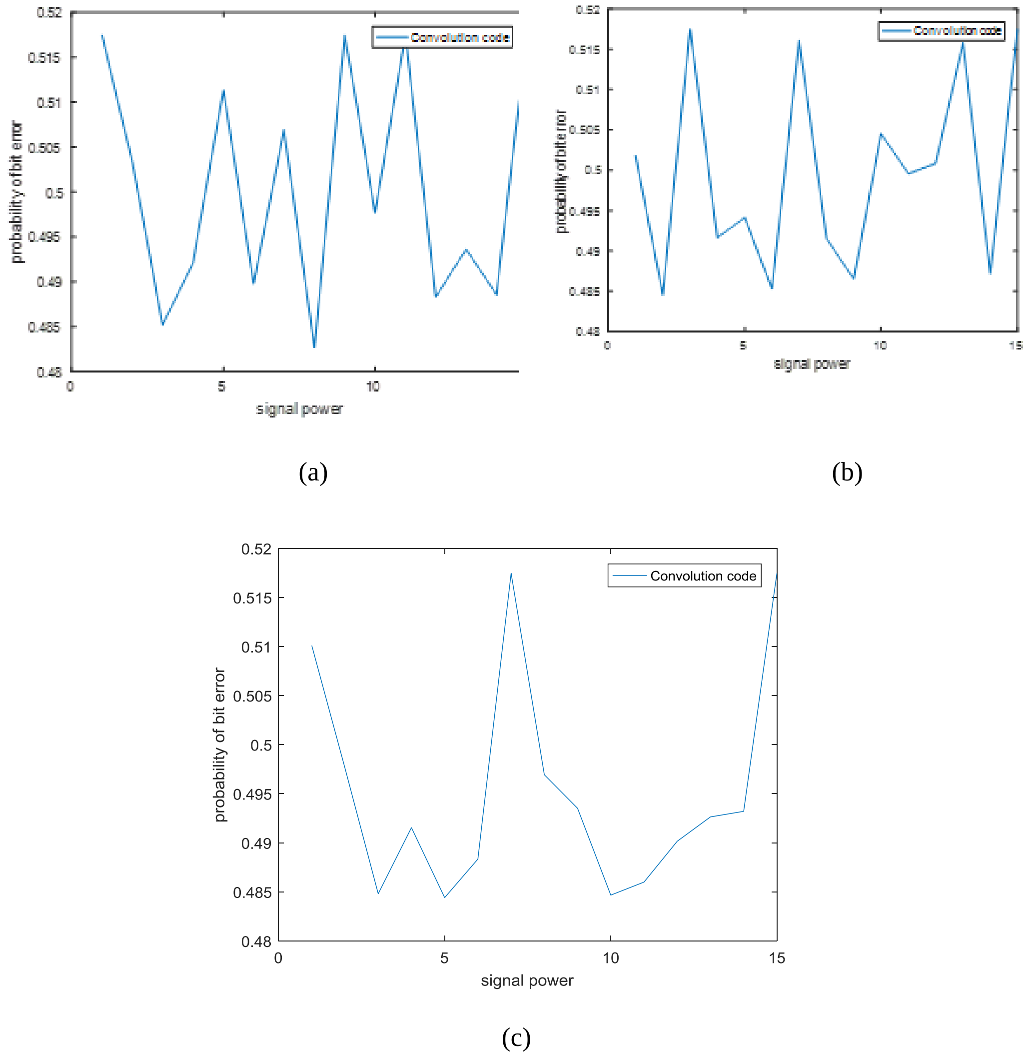


Figure. 17 'Arabic Text' contour data detection using Viterbi decoding algorithm for (a) 16-QAM, (b) 32-QAM, and (c) 64-QAM

The plots show that *NCC* values for reconstructed contours and binary images are identical using analysed algorithm of 16-QAM, 32-QAM, and 64-QAM but by using 16-QAM the image can carry the AWGN more and the consumed time is lower than that of others. The CR using Trapezoid method for 'Tools' image is higher than that for 'Arabic Text' image with accepted level of distortion. However, the selection of thresholding value plays a critical role in the compression process.

10. CONCLUSIONS

The digital communication scheme is simulated (MATLAB Programming version 2016) using spread spectrum with different M-QAM modulation. This paper presents a novel data compression scheme for storage space reduction and higher transmission speed purposes using both arithmetic coding as source coding and Trapezoid method as contour compression in spatial domain. The binary image contours are extracted using SSPCE algorithm and will be transmitted instead of binary image. The compressed contours of the binary image ('Tools') using arithmetic coding and Trapezoid method exceeds to 85.1227% and 17.4067% respectively. To obtain higher compression ratio with small significant visible distortion in the reconstruction quality using Trapezoid method; the accepted level of the reconstruction quality is determined using thresholding. The threshold value is the maximum number of points between each two points in the trapezoid segment shape with the conservation of the accepted level of the reconstruction quality. Then the convolutional coding (Trellis diagram) is using in channel coding before applied direct sequence spreading spectrum. The contour data is scrambling using Arnold transform, which depends on periodicity number of times to give security system performance. For data detection at the receiver, the Viterbi algorithm is used as channel decoding stage via hard decision for probability of bit error computations as final stage at the receiving processes. For binary signal reconstruction with high quality, some morphological operations is performed. The results show that the original binary images can be precisely retrieved from encrypted images and the normalized correlation coefficient (*NCC*) for 'Tools', and 'Arabic Text' images are 0.9753, and 0.9498 respectively. In addition to that the extracted contours at the receiver is identical to the original contour image for both tested images with *NCC* exactly equal to 1 using different 16-32-64-QAM modulation. The proposed algorithm has low complexity and also very useful in real time applications.

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